Subject: Leaving Certificate Maths
Teacher: Mr Murphy
Lesson 4: Algebra II

4.1 Learning Intentions

After this week's lesson you will be able to;

- Carry out long division in algebra
- Solve linear equations with 1/2/3 variables

4.2 Specification

Students learn about	Students working at OL should be able to	In addition, students working at HL should be able to
4.2 Solving equations	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: • $f(x) = g(x)$, with $f(x) = ax+b$, $g(x) = cx+d$ where $a, b, c, d \in \mathbf{Q}$ • $f(x) = g(x)$ with $f(x) = \frac{a}{bx+c} \pm \frac{p}{qx+r}$; $g(x) = \frac{e}{f}$ where $a, b, c, e, f, p, q, r \in \mathbf{Z}$ • $f(x) = k$ with $f(x) = ax^2 + bx + c$ (and not necessarily factorisable) where $a, b, c \in \mathbf{Q}$ and interpret the results – select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to • simultaneous linear equations with two unknowns and interpret the results • one linear equation and one equation of order 2 with two unknowns (restricted to the case where either the coefficient of x or the coefficient of y is $\pm$ 1 in the linear equation) and interpret the results – form quadratic equations given whole number roots	– select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: $f(x) = g(x)$ with $f(x) = \frac{ax+b}{ex+f} \pm \frac{cx+d}{qx+r}$; $g(x) = k$ where $a, b, c, d, e, f, q, r \in \mathbf{Z}$ – use the Factor Theorem for polynomials – select and use suitable strategies (graphic, numeric, algebraic and mental) for finding solutions to • cubic equations with at least one integer root • simultaneous linear equations with three unknowns • one linear equation and one equation of order 2 with two unknowns and interpret the results

4.3 Chief Examiner's Report

Student should develop strategies for checking their answers. In addition to techniques for identifying that an error has been made, techniques for finding those errors quickly and camly, including getting to know one's own weakness, should also be developed.

4.4 Rational Algebra

Addition & Subtraction

The best way to think of the addition and subtraction of fractions in algebra is to follow the same method employed in week 1 with the rational number addition. If you recall these steps were the following:

1) Find the common denominator
2) Divide in each denominator
3) Multiply answer by numerator
4) Repeat for all fractions

Below is the example dealt with in the video, using the video and steps above complete the sum

$$\frac{2x+1}{x}+\frac{3x-1}{5}$$

$$\frac{2x+1}{x-1}-\frac{3x-1}{5x^2+2}$$

Multiplication

Again, dealing with fractions in algebra is the same as the with real numbers. We follow the simple rule of multiplying:

Top by Top
&
Bottom by bottom

$$\frac{3x}{x+4}\times\frac{7x-2}{5x^2}$$

Division

The same applies to division of rational numbers, we treat division of algebraic fractions in the same way as integer fractions.

Turn the fraction you are dividing by upside down and multiply. So, for example:

$$\frac{\frac{3x+1}{x^2-2}}{\frac{4x}{1-4x}} = \frac{3x+1}{x^2-2} \times \frac{1-4x}{4x}$$

$$\frac{x-5}{x+1} \div \frac{x^2-25}{x^2+4x+3}$$

4.5 Long Division in Algebra

Sometimes in algebra we need to use long division, just like in real numbers back in primary school.

One of the main functions of long division is to use one factor of a polynomial in order to find the remaining factors.

For example:

Using long division, find the two remaining factors of:

$$6x^3 + 5x^2 - 10x + 3$$

If one factor is $2x - 1$

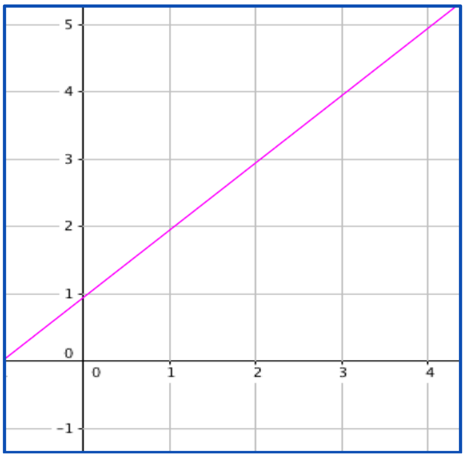
$$6x^3 + 5x^2 - 10x + 3 \div 2x - 1$$

4.6 Linear Equations

A linear equation is an algebraic expression that includes an equals sign.

With the equals sign comes certain rules and consequences. One such consequence to note is that if we graph one of these linear equations on the coordinate plane either by hand or a graphing software program we can see that it generates a straight line.

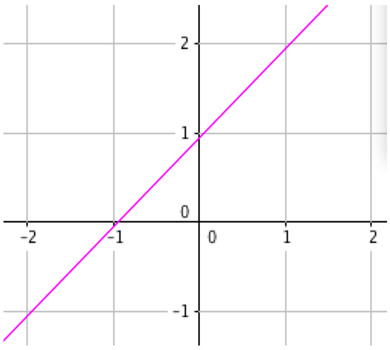
For example, if we graph $y = 1x + 1$ we get the following:



If we are looking to solve one of these equations, we are essentially setting the value of y to be zero. This allows us to solve these equations both graphically and algebraically.

If we are to via graphical methods we need to identify where our linear graph intersects the x-axis (the point with a y-coordinate of zero).

To do this we need to zoom in on our graph



As this has **degree one**, there is **one solution** for x. Which we can see from this graph is – 1.

However, in order to solve this equation with algebraic methods, we need to recall the basics of what an equation is. The best way to describe an equation is through the use of a balance (or weighing scales).

Throughout solving an equation, the scales (equation) must be kept **balanced.**

What this translates to is that whatever changes we make on one side of the scales (equation) must also be made on the other.

In terms of deciding what changes to make, the decision is based on the terms that we wish to move around within the equation. In our equations, we are attempting to solve for x, which means that we need to isolate x on one side of the equation. Therefore, we must move all other terms that are on the same side as x across the equals.

In order to move these terms correctly, we must carry out the **inverse** operation on them. For example, if we wish to get rid of a +4 we would subtract 4 from both sides. Below is a list of operations and their inverses.

Operation	Inverse
+	-
÷	x
$\square^2$	$\sqrt{\square}$
$\log_a \square$	a^b

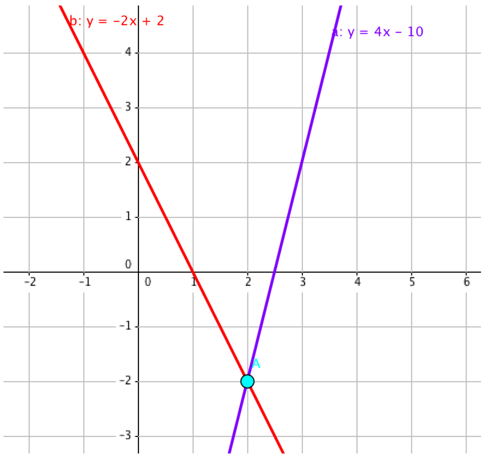
$$y = 1x + 1$$
$$0 = 1x + 1$$
$$0 - 1 = 1x + 1 - 1$$
$$-1 = 1x \text{.....ANS}$$

4.7 Linear Simultaneous Equations

2 variables:

For these equations, we are tasked with solving two linear equations at the same time. Unlike the single linear equation, where we find the point of intersection between our line and the x-axis, we are now finding the point of intersection between the two lines. This can, just like the single equations, be found graphically and algebraically.

Using graphing methods, we get:



From this graph we can see that the P.O.I. is (2,-2). This can be verified using algebraic methods:

$$y = -2x + 2$$
$$y = 4x - 10$$

3 variables:

Simultaneous linear equations with 3 variables arise when we have three linear equations with 3 variables each.

Three variables mean we no longer have a line represented in a 2-D space, it's now 3-D. Therefore, our lines have now become planes.

3 variables mean 3 dimensions
X-axis (left to right)
Y-axis (up and down)
Z-axis (Front to Back)

From the 2 variable example we can see that when 2 lines intersect it is at a point, however with two planes they intersect at a line. Therefore in order to find a point of intersection of the planes we cannot use just 2 planes, we need 3.

From the 2 variable example we can see that when 2 lines intersect it is at a point, however with two planes they intersect at a line. Therefore, in order to find a point of intersection of the planes we cannot use just 2 planes, we need 3.

A pair of planes intersect along a line and then so do a different pair of planes. We then find out where these two lines intersect to find the point of intersection of all three planes.

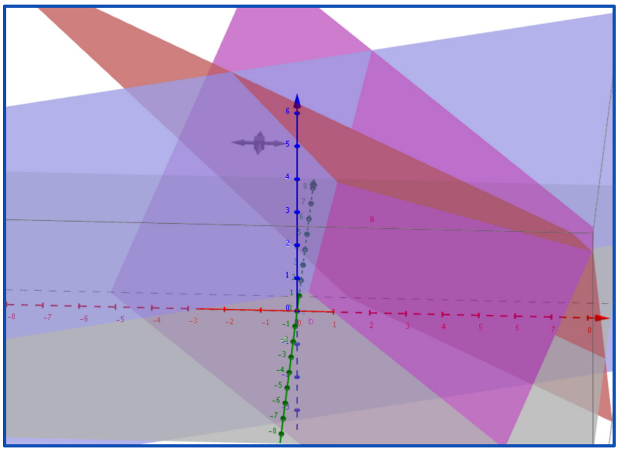
Here are our 3 planes:

$$x + y = z = 6$$

$$5x + 3y - 2z = 5$$

$$3x - 7y + z = -8$$

Which are difficult to represent graphically, but we can see this better on the video.



Now we will look at solving these using algebraic techniques:

$$x + y = z = 6$$

$$5x + 3y - 2z = 5$$

$$3x - 7y + z = -8$$

4.8 Recap of Learning Intentions

After this week's lesson you will be able to;

- Carry out long division in algebra
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4.9 Homework Task

Solve the simultaneous equations;

$$x + y + z = 16$$
$$\frac{5}{2}x + y + 10z = 40$$
$$2x + \frac{1}{2}y + 4z = 21$$

4.10 Solutions to 3.11

Look at the following expressions and factorise/expand.

1) $6b(a^2b^2 - 3abc)$

$6a^2b^3 - 18ab^2c$

2) $(3x^2 - 4xy)(2xy^2 - 4y)$

$6x^3y^2 - 12x^2y - 8x^2y^3 + 16xy^2$

3) $\frac{12x^2yz^2}{2xyz} = 6xz$

4) $\frac{21a^3bc}{14a^4bc} = \frac{3}{2a}$

Factorise:

5) $6xy + 10x^2y$

$2x(3y + 5xy)$

6) $49 - y^2$

$(7)^2 - (y)^2$

$(7 - y)(7 + y)$

7) $x^2 + 20x + 100$

$(x+10)(x+10) = (x+10)^2$

8) $m^2 + 16m + 64$

$(m+8)(m+8) = (m+8)^2$

9) $12t^2 + 11x + 2$

$(3x+2)(4x+1)$